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Radiative corrections to motion of an electron in external electromagnetic fields

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Abstract. The value of the anomalous magnetic moment of an electron as a function depending on the parameters of the external electromagnetic field is obtained. Physical analysis of the obtained results is presented.

1. Introduction

Theoretical papers devoted to pulsars have suggested recently that magnetic fields of the order of the characteristic quantum electrodynamic value $H \gtrsim H_0 = m^2c^3/e\hbar = 4{\cdot}41 \times 10^{13}$ Oe probably exist. Theoretical description of some of the effects observed under such conditions and comparison of the obtained results with astrophysical investigations make it possible to verify conclusions of the charged particle interaction theory in strong electromagnetic fields. This verification is rather interesting because the laboratory realisation of such fields is not possible now.

The well known conclusions of the classical synchrotron radiation theory are essentially modified in the case of these strong fields. Really, from the general formula of electron energy in a magnetic field

$$E = (m^2c^4 + 2neHc\hbar)^{1/2}, \qquad p_3 = 0$$

it is obvious that in the superstrong magnetic field $H > H_0$ the electron is relativistic even on the first excited level $n = 1$. Energy of the radiated photon in dipole transitions from the state $n = 1$ to $n = 0$, practically coincides with the energy of the electron.

In this case expressions for the total power and probability of the radiation process differ from those in classical and quantum ultra-relativistic $(H/H_0 \times E/mc^2 \gg 1)$ limits, which means manifestation of a discrete electron energy spectrum, when the electron is in a state with small values of quantum levels n. Furthermore, when $H \gg H_0$ the probabilities of spontaneous electron transitions from $n = 1$ to $n = 0$ with and without spin re-orientation become equal in order of magnitude.

Let us note that, in fields $H \sim H_0$, when the definite conditions are realised, the quantum electrodynamic effects connected with electron and photon interactions with electromagnetic and electron–positron vacua can be considerably intensified.

The role of vacuum effects in the constant electromagnetic field was discussed for the first time by Schwinger (1948), who gave the theoretical explanation of the anomalous electron magnetic moment. As is known, radiative corrections to electron and photon motion manifest themselves through the alteration of electron mass and the appearance of photon mass in the external field. These corrections are determined by the peculiarities of the particle motion and field intensity.

This paper is intended to present, as fully as possible, results concerning the calculation of the anomalous electron magnetic moment in the external electromagnetic field, and in particular, the dependence of the anomalous electron magnetic moment on the intensity of the magnetic field H and electron energy E. Therefore, the field correction to the electron mass depending on spin will be under consideration. It is the spin part of the electron mass that is connected with the existence of the electron vacuum magnetic moment. It should be noted that various aspects of this problem have been considered by many authors. Here our approach follows Ternov and Tumanov (1960) and Ternov *et al* (1968a, b). The other form of the electron mass operator in the magnetic field, in an α-approximation with respect to the radiation field, was used by Schwinger (1973) and Tsai Wu-Yang and Asim Yildiz (1973) (see also Bayer *et al* 1974).

2. Radiative corrections to mass of an electron in external magnetic fields

The electron mass correction in the magnetic field which is conditioned by electron interaction with the vacuum can be written as follows:

$$W_{\zeta,\zeta'} = \Delta m^{(1)} + \Delta m^{(2)} = \frac{e^2}{4\pi} \sum_{n',\epsilon} \int\int \frac{\kappa \, d\kappa \sin\theta \, d\theta}{K - \epsilon(K'+\kappa)} (F_1(\kappa,\theta) + F_2(\kappa,\theta)),$$

$$F_1(\kappa,\theta) = (D_1 D_1' + D_{-1} D_{-1}')((1 - \epsilon k_0^2/KK')(I^2_{n,n'-1}(z) + I^2_{n-1,n'}(z))$$

$$- \epsilon k_0^2/KK'(I^2_{n-1,n'-1}(z) + I^2_{n,n'}(z)) - 2\epsilon \frac{2\gamma(nn')^{1/2}}{KK'} I_{n,n'}(z) I_{n-1,n'-1}(z)),$$

$$F_2(\kappa,\theta) = (D_{-1}D_{-1}' - D_1 D_1') k_0/K \left((1 - \epsilon K/K')\frac{n-n'}{z} - \epsilon K/K'\right)$$

$$\times (I^2_{n,n'}(z) - I^2_{n-1,n'-1}(z)),$$

$$z = \kappa^2 \sin^2\theta/2\gamma.$$

Here n is a principal quantum number characterising electron energy in the magnetic field,

$$E_n = c\hbar K = c\hbar(k_0^2 + 2\gamma n)^{1/2}, \qquad k_0 = mc/\hbar, \qquad \gamma = eH/c\hbar,$$

$I_{n,n'}(z)$ are Laguerre functions; $\zeta, \zeta' = \pm 1$ determine the dependence of the quantum state on electron spin orientation with respect to the direction of the magnetic field (ζ corresponds to the initial state and ζ' to the final one). Spin coefficients D_1 and D_{-1} satisfy the normalised condition $D_1^2 + D_{-1}^2 = 1$.

It is essential to understand that the spin part of the vacuum correction to an electron energy may be distinguished only for $n \neq 0$. This can be explained by the fact that states with $n \neq 0$ are doubly degenerate towards magnetic field spin projections which split each of these degenerate states. This is why different electron spin projections to the field correspond to contributions, which are different in sign, to the total vacuum energy.

So far as the operation of splitting is relativistically covariant for $n \neq 0$, it is possible to determine the physical meaning of each term of the sum $\Delta m^{(1)} + \Delta m^{(2)}$ in the rest system. As a result, the real part of the electron vacuum energy depending on spin can be interpreted as an interaction energy of the additional electron magnetic moment with the external magnetic field

$$\mathrm{Re}\, \Delta m^{(2)} = (\boldsymbol{\mu H}) = -\frac{\alpha}{2\pi} \mu_0 \zeta H f(n, a), \qquad \mu_0 = \frac{e\hbar}{2mc}.$$

The constant μ is defined by

$$\mu = -\frac{\alpha}{2\pi} \mu_0 f(n, a)$$

and can be identified with the anomalous magnetic moment of the electron. The function $f(n, a)$ is determined by the expression obtained by Ternov *et al* (1968a, b):

$$f(n, a) = -8a \sum_{n'} \int_0^\infty \int_0^{\pi/2} \frac{x\, dx \sin\theta\, d\theta}{[(\xi + x^2\cos^2\theta)^{1/2} + x]^2 - 1} \left(1 + \frac{\xi - 1 + x^2\sin^2\theta}{x\sin^2\theta(\xi + x^2\cos^2\theta)^{1/2}}\right)$$
$$\times (I^2_{n,n'}(z) - I^2_{n-1,n'-1}(z)), \tag{1}$$

where $a = k_0^2/2\gamma = H_0/2H$, $\xi = (n'+a)/(n+a)$, $z = (n+a)x^2\sin^2\theta$. The value $\mathrm{Re}\, \Delta m^{(2)}$ has no divergences and hence its renormalisation is not required.

The case of the ground state ($n = 0$) is special. The electron spin in this case is oriented against the direction of the magnetic field ($\zeta = -1$) only. Therefore the separation of the electron mass into two parts by the same method at $n = 0$ cannot be accomplished now. This state can be characterised by the full correction to the electron mass $\Delta m(n = 0) = -(\alpha/2\pi)(eH/2m)f(0, a)$,

$$f(0, a) = -8a \sum_{n'} \int_0^\infty \int_0^{\pi/2} \frac{x\, dx \sin\theta\, d\theta}{[(\xi' + x^2\cos^2\theta)^{1/2} + x]^2 - 1} \left[\left(\frac{x}{(\xi' + x^2\cos^2\theta)^{1/2}}\right) I^2_{0,n'-1}(z')\right.$$
$$\left. + \left(2 + \frac{2x^2\sin^2\theta + \xi' - 1}{x\sin^2\theta(\xi' + x^2\cos^2\theta)^{1/2}}\right) I^2_{0,n'}(z')\right] \tag{2}$$

where $\xi' = \xi_{n=0} = 1 + n'/a$, $z' = z_{n=0} = ax^2\sin^2\theta$. In the state $n = 0$ the value $\mathrm{Im}\, \Delta m = 0$. The values of $\Delta m^{(1)}$ for $n = 0$, and Δm in the case $n = 0$ have divergences that are removed by a mass renormalisation. It is easy to see that separation of the value Δm into two parts for the state $n = 0$ is not simple, because it is not possible to extract in the correct way two finite values from the value being divergent. This circumstance was emphasised in Ternov *et al* (1968a, b), where the value Δm for the state $n = 0$ was calculated.

Considering the above, we present the final results for functions $f(n, a)$ and $f(0, a)$ in the following regions for parameters n and a (Ternov *et al* 1968a, b):

(i) $a \gg 1 (H \ll H_0)$ and small n ($n \ll a$);

(ii) $a \gg 1$, $n \gg 1$; moreover, values $n \gg a^3$ or $n \ll a^3$ are possible; it is the domain of ultra-relativistic electron energy;

(iii) $a \to 0$ ($H \gg H_0$) and values n are small; in this case an electron is relativistic on any level n.

These kinds of separations are interesting from the physical aspect. Moreover, final formulae for $\Delta m(n = 0)$ and $\mathrm{Re}\, \Delta m^{(2)}$ in these regions can be represented as elementary functions.

The first is a quantum case because numbers of n are small. Formulae for $f(n, a)$ and $f(0, a)$ under $a \gg 1$ and $a \gg n$ can be obtained in the form of decompositions over the parameter $1/a \ll 1$ (Newton 1954, Tsai Wu-Yang and Asim Yildiz 1973, Ternov *et al* 1968a, b):

$$f(n, a) = 1 - \frac{7}{3a^2}\left(\ln a - \frac{576 \ln 2 - 83}{420}\right),$$

$$f(0, a) = 1 - \frac{1}{a}\left(\frac{4}{3}\ln a - \frac{13}{16}\right) - \frac{7}{3a^2}\left(\ln a - \frac{576 \ln 2 - 83}{420}\right).$$

The field corrections to electron mass and anomalous magnetic moment in this case are very small.

Let us consider the behaviour of the function $\mathrm{Re}\, f(n, a)$ in the case when electron energy is ultra-relativistic ($(n/a)^{1/2} \gg 1$ and $a \gg 1$). With these assumptions (Ritus 1969, Bayer *et al* 1971, Sokolov and Ternov 1974):

$$f(n, a) = \frac{3}{\xi_0}\int_0^\infty \frac{u\, du}{(1+u)^3}\int_0^\infty dt \sin\frac{3u}{2\xi_0}\left(t + \frac{t^3}{3}\right). \tag{3}$$

The characteristic feature of this case is the dependence of radiative corrections $\mathrm{Re}\, f(n, a)$ on only one invariant $\xi_0 = \frac{3}{4}(n/a^3)^{1/2} = 3\chi/2$. The corresponding imaginary parts of $\Delta m^{(1)}$ and $\Delta m^{(2)}$, when $(n/a^3)^{1/2} \gg 1$ are also determined by this parameter (Ritus 1969, Bayer *et al* 1971, Sokolov and Ternov 1974). The following expressions for the two limit values of χ can be obtained by integrating (3):

$$\chi \ll 1 \qquad f = 1 - 12\chi^2\left(\ln 1/\chi + C + \frac{\ln 3}{2} - \frac{37}{12}\right) + \dots \tag{4}$$

$$\chi \gg 1 \qquad \frac{\alpha}{2\pi} f = \frac{\alpha\Gamma(1/3)}{(9\sqrt{3})(3\chi)^{2/3}} + \dots; \qquad C = 0{\cdot}577. \tag{5}$$

Thus in the semi-classical region ($\chi \ll 1$) corrections to Schwinger's anomalous magnetic moment of the electron are again small.

On the other hand, in the quantum region $\chi \gg 1$, values of the anomalous magnetic moment are changing significantly; when χ increases the moment decreases as $\chi^{-2/3}$. The functional dependence of the value $(\alpha/2\pi)f$ on parameter χ was found by Ternov *et al* (1968a, b). Formulae (3) and (5) were obtained by Ritus (1969) (see also Bayer *et al* 1971).

A completely new situation with corrections to mass and the anomalous magnetic moment takes place in the case of super-strong magnetic fields ($H \gg H_0$). The question about behaviour of vacuum corrections to mass and anomalous electron magnetic moments in such fields was first considered and solved, as far as we know, by Ternov *et al* (1968a, b). The main result of that paper, concerning the unusual behaviour of the anomalous electron magnetic moment in the fields $H \gg H_0$ was recently confirmed by Bayer and Mil'shtane (1975, 1976).

The asymptotic expressions for functions $f(n, a)$ and $f(0, a)$ can be represented with the double logarithmic precision as $a \to 0$ and $|\ln(1/a)| \gg 1$, that is when the logarithmic value is much higher than unity. In this case, the basic contribution is given by the term in expressions (1), (2) corresponding to the transition on an intermediate level $n' = 0$. If $a \to 0$ from (1), (2) it follows that $f(n, a)$ is given by

(Ternov *et al* 1968a, b, Bayer and Mil'shtane 1975, 1976):

$$f(n, a)=\frac{2a \ln a}{n} \tag{6}$$

and for $f(0, a)$ (Ternov *et al* 1968a, b, Jancovici 1969, Newton 1971):

$$f(0, a)=-2a \ln^2 a. \tag{7}$$

From formula (6) it is easy to see that the anomalous magnetic moment of the electron in the super-strong magnetic field differs considerably from the case of the weak field ($a \gg 1$). The function $f(n, a)$, as $a \to 0$, is negative and therefore the value of the anomalous magnetic moment tends to zero, being permanently negative when a decreases (intensity of the field increases). Function $f(n, a)$ is positive for $a \gg 1$ as well as continuous in the whole region of alteration of its argument (if n is fixed). Therefore, there are at least two points, at one of which $f(n, a)$ is equal to zero while at the other the function minimum is achieved. It is clear that the minimum value is negative and both points are near $a \sim 1$ ($H \sim H_0$).

First-order corrections to the electron mass were considered in accordance with perturbation theory to constant $\alpha = e^2/c\hbar$, with the obtained results valid at $\Delta m \ll m$. This condition restricts the corresponding parameters. In the case of the second region the formula is valid as long as $\alpha\chi^{2/3} < 1$, a condition made by Ritus (1969). In the field $H \gg H_0$ the restriction takes the form

$$1 \ll \ln^2 (1/a) \ll 4\pi/\alpha.$$

In the super-strong magnetic field $H \gg H_0$ the vacuum correction to the electron energy of the ground state is positive, so the gap between spectra of the single particle states in such a field increases compared with the weak field (see also O'Connel 1968). Hence in the magnetic field $H > H_0$, if the energy of photons equals $2mc^2$, the electron–positron pair production cannot take place, generally speaking. In other words, the value $2mc^2$ is the lower limit of the total energy of two photons in vacuum or in the weak magnetic field for the following process $\gamma+\gamma \to e^- + e^+$ if vacuum corrections are ignored.

It is of interest that the asymptotic formulae for $\Delta m(n=0)$ and $f(n, a)$ in the super-strong magnetic field depend on H/H_0 as do renormalisation expressions of free electron form factors $f(t)-1$ and $g(t)$ at $t=(p_2-p_1)^2 \gg 4m^2c^4$ (p_1 and p_2 are the four-momenta of the electron). Assuming the invariant parameter t is equal to the electron kinetic energy in the magnetic field

$$-t = eHc\hbar, \qquad n=0 \tag{8}$$

$$-t = eHc\hbar n, \qquad n \neq 0 \tag{9}$$

in formulae for $f(t)-1$ and $g(t)$, it is easy to see that these expressions coincide with (6) and (7). In fact, relations calculated with the double logarithmic precision for $f(t)-1$ and $g(t)$ in the region $-t \gg 4m^2c^4$ have the form (Lifshitz and Pitaevsky 1971):

$$f(t)-1 = -\frac{\alpha}{2\pi}\left(\frac{1}{2}\ln^2 \left|\frac{t}{m^2c^4}\right| + 2\ln\frac{m}{\lambda}\ln\left|\frac{t}{m^2c^4}\right|\right), \tag{10}$$

$$g(t) = -\frac{\alpha}{\pi}\frac{m^2c^4}{t}\ln\left|\frac{t}{m^2c^4}\right|. \tag{11}$$

Here λ is a virtual photon mass. Substituting (8), (9) in (10), (11) for the terms giving the main contribution results in the expressions

$$f(t)-1=-\frac{\alpha}{4\pi}\ln^2 a=-\frac{\Delta m(n=0)}{m},$$

$$g(t)=\frac{\alpha}{\pi}\frac{a\ln a}{n}=\frac{\alpha}{2\pi}f(n,a)=-\frac{\mu(n\neq 0)}{\mu_0}.$$

This fact can be understood if we compare the contribution of a self-energy electron mass diagram in the magnetic field of high intensity with the contribution of a free electron vertex diagram.

3. Hydrogen-like atoms in strong magnetic fields

It is necessary to take into consideration vacuum corrections to energy in super-strong magnetic fields when researches on atomic energy spectra, particularly for small values of a nucleus of charge z are carried out.

The problem, in connection with an energy spectra calculation of different atoms in a strong magnetic field, was given much consideration (London 1956, Cohen *et al* 1970, Kadomtsev and Kudryavtsev 1971, Kraynov 1973). In particular, Kadomtsev and Kudryavtsev (1971) have investigated the question of the shell structure of heavy atoms in the magnetic field. They showed that in strong magnetic fields, the electron shells of atoms are compressed in the plane perpendicular to the direction of the magnetic field and stretched along the field. So the distribution of the electron density in heavy atoms can violate spherical symmetry.

It is essential that in the case of atoms with small values of z (in particular, for $z=1$) vacuum corrections to the energy of an atomic electron can considerably change the position of the electron ground state.

Dirac's equation for the electron in the magnetic field plus Coulomb field has the most simple form of solutions in cylindrical coordinates ρ, ϕ, z with axis z directed along the magnetic field. This form (Sokolov and Ternov 1974) is given by

$$\psi=\exp\left(-\mathrm{i}\epsilon\frac{E}{\hbar}t+\mathrm{i}(l-\tfrac{1}{2})\phi\right)\frac{1}{2\pi}\begin{pmatrix} f_1(\xi)\,\mathrm{e}^{-\mathrm{i}\phi/2} \\ f_2(\xi)\,\mathrm{e}^{+\mathrm{i}\phi/2} \\ f_3(\xi)\,\mathrm{e}^{-\mathrm{i}\phi/2} \\ f_4(\xi)\,\mathrm{e}^{+\mathrm{i}\phi/2} \end{pmatrix},$$

where $\xi=(\rho^2+z^2)^{1/2}$, and $\epsilon=\pm 1$ is the sign coefficient. The functions f_i are determined by equations

$$\begin{aligned}&\left(\epsilon K\mp k_0-\frac{V(\xi)}{c\hbar}\right)f_{1,3}+\mathrm{i}R_2 f_{2,4}+\mathrm{i}\frac{\partial}{\partial z}f_{3,1}=0\\ &\left(\epsilon K\mp k_0-\frac{V(\xi)}{c\hbar}\right)f_{2,4}+\mathrm{i}R_1 f_{3,1}-\mathrm{i}\frac{\partial}{\partial z}f_{2,4}=0.\end{aligned}\qquad(12)$$

Operators R_1 and R_2 may be written in the form

$$R_1=\sqrt{\gamma\eta}\left(2\frac{\mathrm{d}}{\mathrm{d}\eta}-1-\frac{l-1}{\eta}\right),\qquad R_2=\sqrt{\gamma\eta}\left(2\frac{\mathrm{d}}{\mathrm{d}\eta}+1+\frac{l}{\eta}\right).$$

Here also the following relations hold:

$$V(\xi) = -ze^2/\xi, \qquad \gamma = eH/2c\hbar, \qquad \eta = \gamma\rho^2, \qquad K = E/c\hbar.$$

Let us assume that the magnetic field intensity is so high that the radius of the electron orbit $R = ((n+\frac{1}{2})\gamma)^{1/2}$ is much less than Bohr's radius $r_B = \hbar^2/me^2$. In this case the motion of the electron on the plane xy is determined mainly by the influence of the magnetic field and hence the solution of equation (12) can be reached in the form

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \sqrt{2\gamma} \begin{pmatrix} C_1 I_{n-1,s}(\rho)\chi_1(z) \\ C_2 I_{n,s}(\rho)\chi_2(z) \\ C_3 I_{n-1,s}(\rho)\chi_3(z) \\ C_4 I_{n,s}(\rho)\chi_4(z) \end{pmatrix} \tag{13}$$

where $I_{n,s}(\rho)$ are the Laguerre functions.

The correction to the ground state energy of the electron in the magnetic field is now characterised by the Coulomb nuclear field. The electron motion along the direction H in the presence of the Coulomb field changes essentially. In this case the electron moves in the finite region and its energy spectrum becomes discrete. For the ground state ($n = 0$) functions $f_1 = f_3 = 0$, according to properties of the Laguerre functions, and $f_{2,4}$ are determined by the equations

$$-\mathrm{i}\frac{\partial}{\partial z} f_4 + (\epsilon K - k_0 - \tilde{V}) f_2 = 0$$

$$-\mathrm{i}\frac{\partial}{\partial z} f_2 + (\epsilon K + k_0 - \tilde{V}) f_4 = 0.$$

Writing the electron energy in the form $K = k_0 - \tilde{\epsilon}$, where $\tilde{\epsilon} \ll k_0$ after calculations in accordance with the method shown by Landau and Lifshitz (1974) for the energy of the ground state, we have

$$E = mc^2(1 - \tfrac{1}{2}\alpha^2 \ln H/H_0\alpha^2). \tag{14}$$

We have to note that formula (14) is valid with the logarithmic precision.

The vacuum correction to the energy of the electron ground state in the magnetic field $H \gg H_0$ is

$$E_{\mathrm{vac}} = \frac{\alpha}{2\pi}\frac{mc^2}{2}\left(\ln\frac{2H}{H_0}\right)^2.$$

Comparing the last formula with (14) it can be seen that at $H \gg H_0$ the vacuum correction to electron energy may be of the same order or greater in comparison with the contribution of the electron interaction with a Coulomb field of a nucleus, particularly, for $z \sim 1$. It should be noted that the vacuum correction and Coulomb energy have opposite signs.

The exact expression for the ground state energy of a hydrogen-like atom in the strong magnetic field was obtained by Kraynov (1973) in the form

$$E_1 = \cos\{z\alpha \ln [H/H_0(1-\epsilon_1^2)^{1/2}]\}, \qquad \epsilon_1 = \tilde{\epsilon}/mc^2.$$

The region of applicability of this relation as shown by Cohen *et al* (1971), is limited by inequalities

$$H \gg H_0 \qquad \text{and} \qquad H \ll 2\pi H_0/\alpha. \tag{15}$$

The last restriction is connected with taking into consideration the anomalous electron magnetic moment interaction with the magnetic field. The value of the anomalous magnetic moment in this case was assumed to be equal to the constant $\mu_0\alpha/2\pi$ independently of the field intensity. However, in reality inequality (15) is not required as it follows from the analysis given above.

4. The anomalous magnetic moment of electrons in the presence of the electromagnetic plane wave

The new situation concerning field corrections to an electron mass arises when the mass electron operator in the homogeneous constant magnetic field and the electromagnetic plane wave field (Rodionov *et al* 1976) are studied. This superposition of fields is of interest due to the presence of the constant magnetic field. It permits consideration of spin effects in the electromagnetic plane wave field combined with the effects in the magnetic field. As a result the contribution to the anomalous electron magnetic moment due to the action of the electromagnetic plane wave may be obtained. In this case the value of the anomalous electron magnetic moment is a function of parameters connected with the whole field, in particular, amplitude and frequency of the wave ($\mathscr{E}_0$, ω), intensity of the magnetic field (H) and also the electron energy.

Corrections to Schwinger's value of the anomalous electron magnetic moment in such a field configuration are written in the form

$$\mu = \frac{\alpha}{2\pi}\mu_0\xi^2\left(-1+\frac{1+3\kappa^2}{(1-\kappa^2)^2}+\frac{2\kappa^4+6\kappa^6}{(1-\kappa^2)^3}\ln\kappa\right).$$

Here invariant values ξ and κ are expressed by relations

$$\xi = \frac{e\mathscr{E}_0}{mc\omega} \ll 1, \qquad \kappa = \frac{2\lambda\omega\hbar}{(mc^2)^2}, \qquad \lambda = E - cp_3.$$

In cases $\kappa \ll 1$ and $\kappa \gg 1$, for $\mu(\xi, \kappa)$, the following expressions may be obtained:

$$\kappa \ll 1 \qquad \mu = -\mu_0\xi^2\frac{3\alpha}{\pi}\kappa^2\ln\frac{1}{\kappa},.$$

$$\kappa \gg 1 \qquad \mu = -\mu_0\xi^2\frac{\alpha}{2\pi}\left(1+2\frac{\ln\kappa}{\kappa^2}\right).$$

Let us note that corrections to the anomalous magnetic moment due to influence of the wave are square to the wave amplitude $\mathscr{E}_0$. The total correction to the value of the anomalous magnetic moment of an electron moving in the superposition of the constant magnetic field and the field of the electromagnetic plane wave in the semi-classical approximation ($\chi \ll 1$) and at $\kappa \gg 1$ is given by:

$$\mu = \mu_0\frac{\alpha}{2\pi}\left[1-12\left(\chi^2\ln\frac{1}{\chi}+2\chi_1^2\ln\frac{1}{\kappa}\right)\right],$$

where $\chi_1 = (\mathscr{E}_0/H_0)(\lambda/mc^2)$. This formula is valid at $\ln(1/\chi) \gg 1$ and $\ln(1/\kappa) \gg 1$. It can be verified experimentally because already it seems possible to measure the correction χ_1^2 to Schwinger's anomalous magnetic moment of the electron $(\alpha/2\pi)\mu_0$ in experiments with laser beams, whereas measuring of the correction χ^2 for the laboratory constant magnetic fields is not yet possible.

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